

練習 1 5

- (1) $x^2 - 8x + 16 = x^2 - 2 \cdot x \cdot 4 + 4^2$
 $= (x - 4)^2$
- (2) $4x^2 + 28xy + 49y^2 = (2x)^2 + 2 \cdot 2x \cdot 7y + (7y)^2$
 $= (2x + 7y)^2$
- (3) $9a^2 - 48ab + 64b^2 = (3a)^2 - 2 \cdot 3a \cdot 8b + (8b)^2$
 $= (3a - 8b)^2$
- (4) $16x^2 - 25y^2 = (4x)^2 - (5y)^2$
 $= (4x + 5y)(4x - 5y)$
- (5) $x^2 + 6x + 8 = x^2 + (2 + 4)x + 2 \cdot 4$
 $= (x + 2)(x + 4)$
- (6) $x^2 - 5xy + 6y^2 = x^2 + (-2y - 3y)x + (-2y) \cdot (-3y)$
 $= (x - 2y)(x - 3y)$
- (7) $x^2 + xy - 12y^2 = x^2 + (-3y + 4y)x + (-3y) \cdot 4y$
 $= (x - 3y)(x + 4y)$
- (8) $x^2 - 2ax - 15a^2 = x^2 + (3a - 5a)x + 3a \cdot (-5a)$
 $= (x + 3a)(x - 5a)$

練習 1 6

- (1) $2x^2 + 3x + 1 = (x + 1)(2x + 1)$
- (2) $4x^2 - 15x + 9 = (x - 3)(4x - 3)$
- (3) $6x^2 - 5x - 6 = (2x - 3)(3x + 2)$
- (4) $3x^2 - 2xy - y^2 = (x - y)(3x + y)$
- (5) $3a^2 - 14ab + 8b^2 = (a - 4b)(3a - 2b)$
- (6) $4x^2 + 7ax - 2a^2 = (x + 2a)(4x - a)$

練習 1 7

- (1) $x^2 - y^2 + 6y - 9 = x^2 - (y^2 - 6y + 9)$
 $= x^2 - (y - 3)^2$
 $= [x + (y - 3)][x - (y - 3)]$
 $= (x + y - 3)(x - y + 3)$
- (2) $x^2 - 4x + 4 - 9y^2 = (x^2 - 4x + 4) - 9y^2$
 $= (x - 2)^2 - (3y)^2$
 $= [(x - 2) + 3y][(x - 2) - 3y]$
 $= (x + 3y - 2)(x - 3y - 2)$

練習 1 8

- (1) $x^4 - 5x^2 + 4 = (x^2)^2 - 5x^2 + 4$
 $= (x^2 - 1)(x^2 - 4)$
 $= (x + 1)(x - 1)(x + 2)(x - 2)$
- (2) $x^4 - 81 = (x^2)^2 - 9^2$
 $= (x^2 + 9)(x^2 - 9)$
 $= (x^2 + 9)(x + 3)(x - 3)$
- (3) $(x^2 + 3x)^2 - 2(x^2 + 3x) - 8$
 $= [(x^2 + 3x) + 2][(x^2 + 3x) - 4]$
 $= (x^2 + 3x + 2)(x^2 + 3x - 4)$
 $= (x + 1)(x + 2)(x - 1)(x + 4)$

練習 1 4

- (1) $2x^2y - 6xy^2 + 10xyz = 2xy \cdot x - 2xy \cdot 3y + 2xy \cdot 5z$
 $= 2xy(x - 3y + 5z)$
- (2) $4xy^2z - x^2yz^2 + 2xyz = xyz \cdot 4y - xyz \cdot xz + xyz \cdot 2$
 $= xyz(4y - xz + 2)$
- (3) $a(x - y) - bx + by = a(x - y) - b(x - y)$
 $= (x - y)(a - b)$
- (4) $y(5x - 3) + 2(3 - 5x) = y(5x - 3) - 2(5x - 3)$
 $= (5x - 3)(y - 2)$

練習 1 9

- (1) $x^2 - yz + zx - y^2 = (x - y)z + (x^2 - y^2)$
 $= (x - y)z + (x + y)(x - y)$
 $= (x - y)(x + y + z)$

- (2) $9b - 9 - 3ab + a^2 = -3b(a-3) + (a^2-9)$
 $= -3b(a-3) + (a+3)(a-3)$
 $= (a-3)(a-3b+3)$
- (3) $2x^2 + 6xy + x - 3y - 1 = 3y(2x-1) + (2x^2 + x - 1)$
 $= 3y(2x-1) + (x+1)(2x-1)$
 $= (2x-1)(x+3y+1)$

練習 2 0

- (1) $x^2 + 3xy + 2y^2 + 2x + 5y - 3$
 $= x^2 + (3y+2)x + (2y^2 + 5y - 3)$
 $= x^2 + (3y+2)x + (y+3)(2y-1)$
 $= (x+(y+3))(x+(2y-1))$
 $= (x+y+3)(x+2y-1)$
- (2) $3x^2 - xy - 2y^2 + 6x - y + 3$
 $= 3x^2 - (y-6)x - (2y^2 + y - 3)$
 $= 3x^2 - (y-6)x - (y-1)(2y+3)$
 $= (x-(y-1))(3x+(2y+3))$
 $= (x-y+1)(3x+2y+3)$

練習 2 1

- $ab(a-b) + bc(b-c) + ca(c-a)$
 $= a^2b - ab^2 + ac^2 - a^2c + bc(b-c)$
 $= (b-c)a^2 - (b^2 - c^2)a + bc(b-c)$
 $= (b-c)a^2 - (b+c)(b-c)a + bc(b-c)$
 $= (b-c)\{a^2 - (b+c)a + bc\}$
 $= -(a-b)(b-c)(c-a)$

(p.20) 発展 練習 1

- (1) $(x+1)^3 = x^3 + 3x^2 + 3x + 1$
- (2) $(x-2)^3 = x^3 - 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 - 2^3$
 $= x^3 - 6x^2 + 12x - 8$
- (3) $(3a+b)^3 = (3a)^3 + 3 \cdot (3a)^2 \cdot b + 3 \cdot 3a \cdot b^2 + b^3$
 $= 27a^3 + 27a^2b + 9ab^2 + b^3$
- (4) $(2x-3y)^3 = (2x)^3 - 3 \cdot (2x)^2 \cdot 3y + 3 \cdot 2x \cdot (3y)^2 - (3y)^3$
 $= 8x^3 - 36x^2y + 54xy^2 - 27y^3$

(p.21) 発展 練習 2

- $(a+b)(a^2 - ab + b^2)$
 $= a(a^2 - ab + b^2) + b(a^2 - ab + b^2)$
 $= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 = a^3 + b^3$
 $(a-b)(a^2 + ab + b^2)$
 $= a(a^2 + ab + b^2) - b(a^2 + ab + b^2)$
 $= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 = a^3 - b^3$
- 別解** (後半) $(a+b)(a^2 - ab + b^2) = a^3 + b^3$ において、
 b を $-b$ におき換えると
 $\{a+(-b)\}[a^2 - a(-b) + (-b)^2] = a^3 + (-b)^3$
すなわち $(a-b)(a^2 + ab + b^2) = a^3 - b^3$

(p.21) 発展 練習 3

- (1) $(x+2)(x^2 - 2x + 4) = (x+2)(x^2 - x \cdot 2 + 2^2)$
 $= x^3 + 2^3 = x^3 + 8$
- (2) $(x-3)(x^2 + 3x + 9) = (x-3)(x^2 + x \cdot 3 + 3^2)$
 $= x^3 - 3^3 = x^3 - 27$
- (3) $(3x+y)(9x^2 - 3xy + y^2)$
 $= (3x+y)(3x)^2 - 3x \cdot y + y^2$
 $= (3x)^3 + y^3 = 27x^3 + y^3$
- (4) $(2a-3b)(4a^2 + 6ab + 9b^2)$
 $= (2a-3b)\{(2a)^2 + 2a \cdot 3b + (3b)^2\}$
 $= (2a)^3 - (3b)^3 = 8a^3 - 27b^3$
- (p.21) 発展 練習 4
- (1) $x^3 + 27 = x^3 + 3^3$
 $= (x+3)(x^2 - 3x + 9)$
- (2) $x^3 - 64 = x^3 - 4^3$
 $= (x-4)(x^2 + 4x + 16)$
- (3) $8x^3 - 125y^3 = (2x)^3 - (5y)^3$
 $= (2x-5y)(4x^2 + 10xy + 25y^2)$

問題 (p.22)

問題 1

- (1) 与式
 $= (3x^2 - 2x - 4) + (2x^3 - 2x + 4) + (-3x^2 + 3x - 12)$
 $= 2x^3 + (3-3)x^2 + (-2-2+3)x + (-4+4-12)$
 $= 2x^3 - x - 12$
- (2) 与式 $= (x^2 + 2xy + y^2) + (x^2 - 2xy + y^2)$
 $= 2x^2 + 2y^2$
- (3) 与式 $= (x^2 + 2xy + y^2) - (x^2 - 2xy + y^2) = 4xy$

別解 (1)

$$\begin{array}{r} 2x^3 \qquad \qquad 3x^2 - 2x - 4 \\ \qquad \qquad \qquad -2x + 4 \\ + \qquad \qquad \qquad -3x^2 + 3x - 12 \\ \hline 2x^3 \qquad \qquad \qquad -x - 12 \end{array}$$

問題 2

- (1) 与式 $= 10a^2 - 19ab - 56b^2$
- (2) 与式 $= (2x^2)^2 - 2 \cdot 2x^2 \cdot y + y^2$
 $= 4x^4 - 4x^2y + y^2$
- (3) 与式 $= \{(3a+b) - 2c\}^2$
 $= (3a+b)^2 - 2(3a+b) \cdot 2c + (2c)^2$
 $= 9a^2 + 6ab + b^2 - 12ac - 4bc + 4c^2$
 $= 9a^2 + b^2 + 4c^2 + 6ab - 4bc - 12ca$
- (4) 与式 $= \{(x^2 - 2x) - 1\} \{(x^2 - 2x) - 2\}$
 $= (x^2 - 2x)^2 - 3(x^2 - 2x) + 2$
 $= x^4 - 4x^3 + 4x^2 - 3x^2 + 6x + 2$
 $= x^4 - 4x^3 + x^2 + 6x + 2$
- (5) 与式 $= \{(a-b)(a+b)\}(a^2 + b^2)$
 $= (a^2 - b^2)(a^2 + b^2)$
 $= a^4 - b^4$

(6) 与式 $= (x^4 + 1)(x^2 + 1)((x + 1)(x - 1))$
 $= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
 $= (x^4 + 1)(x^4 - 1) = x^8 - 1$

(7) 与式 $= ((x - 4)(x + 4))((x - 1)(x + 1))$
 $= (x^2 - 16)(x^2 - 1)$
 $= x^4 - 17x^2 + 16$

(8) 与式 $= ((x + 4)(x - 3))((x + 2)(x - 1))$
 $= ((x^2 + x) - 12)((x^2 + x) - 2)$
 $= (x^2 + x)^2 - 14(x^2 + x) + 24$
 $= x^4 + 2x^3 - 13x^2 - 14x + 24$

問題 3

(1)
$$\begin{array}{r} 1 \quad 3 \rightarrow 9 \\ 3 \quad 1 \rightarrow 1 \\ \hline 3 \quad 3 \quad 10 \end{array}$$

$3a^2 + 10a + 3 = (a + 3)(3a + 1)$

(2)
$$\begin{array}{r} 1 \quad -6 \rightarrow -48 \\ 8 \quad -3 \rightarrow -3 \\ \hline 8 \quad 18 \quad -51 \end{array}$$

$8x^2 - 51x + 18 = (x - 6)(8x - 3)$

(3)
$$\begin{array}{r} 3 \quad 4y \rightarrow 20y \\ 5 \quad -6y \rightarrow -18y \\ \hline 15 \quad -24y^2 \quad 2y \end{array}$$

$15x^2 + 2xy - 24y^2 = (3x + 4y)(5x - 6y)$

(4) $9x^2 - 30ax - 24a^2 = 3(3x^2 - 10ax - 8a^2)$

$$\begin{array}{r} 1 \quad -4a \rightarrow -12a \\ 3 \quad 2a \rightarrow 2a \\ \hline 3 \quad -8a^2 \quad -10a \end{array}$$

$9x^2 - 30ax - 24a^2 = 3(3x^2 - 10ax - 8a^2)$
 $= 3(x - 4a)(3x + 2a)$

問題 4

(1) 与式 $= 2x(x^2 - 6xy + 9y^2) = 2x(x - 3y)^2$

(2) 与式 $= (x^2 - 2xy + y^2) - 4$
 $= (x - y)^2 - 2^2$
 $= (x - y + 2)(x - y - 2)$

(3) 与式 $= (4x^2 - 4xy + y^2) - z^2$
 $= (2x - y)^2 - z^2$
 $= (2x - y + z)(2x - y - z)$

(4) 与式 $= (x^2 + 1)(x^2 - 4) = (x^2 + 1)(x + 2)(x - 2)$

(5) 与式 $= ((ac + bd) + (ad + bc))((ac + bd) - (ad + bc))$
 $= (a(c + d) + b(c + d))(a(c - d) - b(c - d))$
 $= (a + b)(c + d)(a - b)(c - d)$
 与式

別解

$= (a^2c^2 + 2acbd + b^2d^2) - (a^2d^2 + 2adbc + b^2c^2)$
 $= a^2(c^2 - d^2) - b^2(c^2 - d^2)$
 $= (a^2 - b^2)(c^2 - d^2)$
 $= (a + b)(a - b)(c + d)(c - d)$

問題 5

(1) 与式 $= (x^3 - 2x^2y) + (xy - 2y^2)$
 $= x^2(x - 2y) + y(x - 2y)$
 $= (x^2 + y)(x - 2y)$

(2) 与式 $= x^2 + (2y - 5)x - (3y^2 - y - 4)$
 $= x^2 + (2y - 5)x - (y + 1)(3y - 4)$
 $= (x - (y + 1))(x + (3y - 4))$
 $= (x - y - 1)(x + 3y - 4)$

$$\begin{array}{r} 1 \quad \times \quad - (y + 1) \rightarrow -y - 1 \\ 1 \quad \times \quad 3y - 4 \rightarrow 3y - 4 \\ \hline 1 \quad - (y + 1)(3y - 4) \quad 2y - 5 \end{array}$$

(3) 与式 $= 2x^2 + (8a - 1)x + (6a^2 + a - 1)$
 $= 2x^2 + (8a - 1)x + (2a + 1)(3a - 1)$
 $= (x + (3a - 1))(2x + (2a + 1))$
 $= (x + 3a - 1)(2x + 2a + 1)$

$$\begin{array}{r} 1 \quad \times \quad 3a - 1 \rightarrow 6a - 2 \\ 2 \quad \times \quad 2a + 1 \rightarrow 2a + 1 \\ \hline 2 \quad (3a - 1)(2a + 1) \quad 8a - 1 \end{array}$$

(4) 与式 $= (a + (b + c))((b + c)a + bc) - abc$
 $= (b + c)a^2 + ((b + c)^2 + bc)a + bc(b + c) - abc$
 $= (b + c)a^2 + (b + c)^2a + bc(b + c)$
 $= (b + c)(a^2 + (b + c)a + bc)$
 $= (b + c)(a + b)(a + c)$
 $= (a + b)(b + c)(c + a)$

問・練習 (第2節)

練習 2 2

(1) $\frac{1}{3} = 0.333\cdots = 0.\dot{3}$

(2) $\frac{7}{33} = 0.2121\cdots = 0.\dot{2}\dot{1}$

(3) $\frac{55}{54} = 1.0185185\cdots = 1.0\dot{1}8\dot{5}$

(4) $-\frac{22}{7} = -3.142857142857\cdots = -3.\dot{1}4\dot{2}8\dot{5}7$

練習 2 3

(1) $x = 0.\dot{1}$ とおくと

$10x = 1.\dot{1}\cdots$
 $-) \quad x = 0.\dot{1}\cdots$
 $9x = 1$

よって $x = \frac{1}{9}$

(2) $x = 0.\dot{1}\dot{2}$ とおくと

$100x = 12.1212\cdots$
 $-) \quad x = 0.1212\cdots$
 $99x = 12$

よって $x = \frac{12}{99} = \frac{4}{33}$

(3) $x = 0.\dot{6}48$ とおくと

$$1000x = 648.648648\cdots$$

$$-) \quad x = 0.648648\cdots$$

$$\hline 999x = 648$$

$$\text{よって} \quad x = \frac{648}{999} = \frac{24}{37}$$

(4) $x = 6.\dot{5}4$ とおくと

$$100x = 654.5454\cdots$$

$$-) \quad x = 6.5454\cdots$$

$$\hline 99x = 648$$

$$\text{よって} \quad x = \frac{648}{99} = \frac{72}{11}$$