3. 三角関数の性質

式の成り立ち.意味を理解しましょう!! ムダ暗記はナンセンス!!

1 $\theta + 2n\pi$

角$\theta+2n\pi$の動径 → 角θの動径と一致

例.

$\sin\boxed{\frac{13}{6}\pi} = \sin\boxed{\frac{\pi}{6}} = \frac{1}{2}$

$\frac{13}{6}\pi = \boxed{\frac{\pi}{6}} + 2\pi$

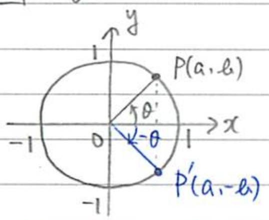
$\cos\boxed{\frac{25}{6}\pi} = \cos\boxed{\frac{\pi}{6}} = \frac{\sqrt{3}}{2}$

$\frac{25}{6}\pi = \boxed{\frac{\pi}{6}} + 4\pi$

$$\boxed{\begin{aligned} \sin(\theta+2n\pi) &= \sin\theta \\ \cos(\theta+2n\pi) &= \cos\theta \\ \tan(\theta+2n\pi) &= \tan\theta \end{aligned}}$$

⇨ Ex.11

2 $-\theta$



$\sin\theta = b$ より

$\sin(-\theta) = -b = -\sin\theta$

$\cos\theta = a$ より

$\cos(-\theta) = a = \cos\theta$

$\tan(-\theta) = \frac{-b}{a}$

$= -\frac{b}{a} = -\tan\theta$

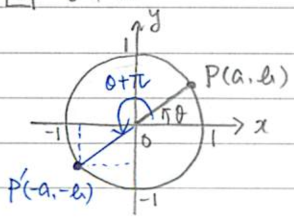
$\tan\theta = \frac{b}{a}$

$$\boxed{\begin{aligned} \sin(-\theta) &= -\sin\theta \\ \cos(-\theta) &= \cos\theta \\ \tan(-\theta) &= -\tan\theta \end{aligned}}$$

例. $\sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

⇨ Ex.12

3 $\theta + \pi$



$\sin\theta = b$

$\sin(\theta+\pi) = -b = -\sin\theta$

$\cos\theta = a$

$\cos(\theta+\pi) = -a = -\cos\theta$

$\tan(\theta+\pi) = \frac{-b}{-a}$

$= \frac{b}{a} = \tan\theta$

$\tan\theta = \frac{b}{a}$

$$\boxed{\begin{aligned} \sin(\theta+\pi) &= -\sin\theta \\ \cos(\theta+\pi) &= -\cos\theta \\ \tan(\theta+\pi) &= \tan\theta \end{aligned}}$$

例 ① $\sin\boxed{\frac{7}{6}\pi} = -\sin\boxed{\frac{\pi}{6}} = -\frac{1}{2}$

$\frac{7}{6}\pi = \boxed{\frac{\pi}{6}} + \pi$

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② $\sin\left(-\frac{10}{3}\pi\right) = -\sin\boxed{\frac{10}{3}\pi}$　　$\frac{10}{3}\pi = \frac{4}{3}\pi + 2\pi$

$= -\sin\boxed{\frac{4}{3}\pi}$　　$\frac{4}{3}\pi = \frac{\pi}{3} + \pi$

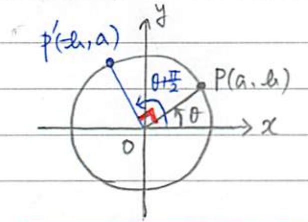
$= -\left(-\sin\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$ //

⇨ Ex. 13

4 $\theta + \frac{\pi}{2}$



$\boxed{\cos\theta = a}$

$\sin\left(\theta+\frac{\pi}{2}\right) = a = \cos\theta$

$\boxed{\sin\theta = b}$

$\cos\left(\theta+\frac{\pi}{2}\right) = -b = -\sin\theta$

$\tan\left(\theta+\frac{\pi}{2}\right) = \frac{a}{-b}$

$= \frac{1}{-\frac{b}{a}} = -\frac{1}{\tan\theta}$

$\boxed{\tan\theta = \frac{b}{a}}$

$$\boxed{\begin{aligned}\sin\left(\theta+\frac{\pi}{2}\right) &= \cos\theta\\ \cos\left(\theta+\frac{\pi}{2}\right) &= -\sin\theta\\ \tan\left(\theta+\frac{\pi}{2}\right) &= -\frac{1}{\tan\theta}\end{aligned}}$$

例 ① $\cos\boxed{\frac{5}{6}\pi} = -\sin\boxed{\frac{\pi}{3}} = -\frac{\sqrt{3}}{2}$ //

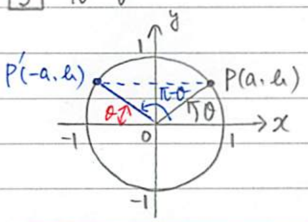
$\frac{5}{6}\pi = \boxed{\frac{\pi}{3}} + \frac{\pi}{2}$

② $\tan\boxed{\frac{2}{3}\pi} = -\frac{1}{\tan\boxed{\frac{\pi}{6}}} = \frac{1}{-\frac{1}{\sqrt{3}}} = -\sqrt{3}$ //

$\frac{2}{3}\pi = \boxed{\frac{\pi}{6}} + \frac{\pi}{2}$

1～4 を使えば、すべての角の三角関数が、第1象限の三角関数で表せる。

5 $\pi - \theta$



$\boxed{\sin\theta = b}$

$\sin(\pi-\theta) = b = \sin\theta$

$\boxed{\cos\theta = a}$

$\cos(\pi-\theta) = -a = -\cos\theta$

$\tan(\pi-\theta) = \frac{b}{-a}$

$= -\frac{b}{a} = -\tan\theta$

$\boxed{\tan\theta = \frac{b}{a}}$

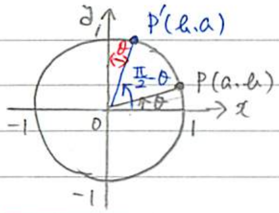
$$\boxed{\begin{aligned}\sin(\pi-\theta) &= \sin\theta\\ \cos(\pi-\theta) &= -\cos\theta\\ \tan(\pi-\theta) &= -\tan\theta\end{aligned}}$$

例 $\cos\boxed{\frac{2}{3}\pi} = -\cos\boxed{\frac{\pi}{3}} = -\frac{1}{2}$ //

$\frac{2}{3}\pi = \pi - \boxed{\frac{\pi}{3}}$

5, 6 は数Ⅰでも扱いましたね。これらは一般角でも使えることを知って欲しいので紹介！

$\boxed{6}$ $\frac{\pi}{2}-\theta$



$\cos\theta = a$

$$\sin\left(\frac{\pi}{2}-\theta\right) = a = \cos\theta$$

$\sin\theta = b$

$$\cos\left(\frac{\pi}{2}-\theta\right) = b = \sin\theta$$

$\tan\theta = \frac{b}{a}$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \frac{a}{b} = \frac{1}{\tan\theta}$$

$$\boxed{\begin{aligned}\sin\left(\frac{\pi}{2}-\theta\right) &= \cos\theta\\ \cos\left(\frac{\pi}{2}-\theta\right) &= \sin\theta\\ \tan\left(\frac{\pi}{2}-\theta\right) &= \frac{1}{\tan\theta}\end{aligned}}$$

[1]～[5] に [6] を組みあわせると、すべての角の三角関数が、0から $\frac{\pi}{4}$ までの三角関数で表せる。

例　$\cos\boxed{\frac{\pi}{3}} = \sin\boxed{\frac{\pi}{6}} = \frac{1}{2}$

$\frac{\pi}{3} = \frac{\pi}{2} - \boxed{\frac{\pi}{6}}$

⇨ Ex. 14

問．次の式の値を求めよ．

① $\sin\left(\theta+\frac{\pi}{2}\right) + \sin(\theta+\pi) + \sin\left(\theta+\frac{3}{2}\pi\right) + \sin(\theta+2\pi)$

② $\alpha = \frac{\pi}{9}$ のとき　$\cos 2\alpha + \cos 4\alpha + \cos 5\alpha + \cos 7\alpha$

$\sin(\theta+2\pi) = \sin\theta$

①解）（与式）$= \sin\left(\theta+\frac{\pi}{2}\right) + \sin(\theta+\pi) + \sin\left(\boxed{\theta+\frac{\pi}{2}}+\pi\right) + \sin\theta$

$\frac{3}{2}\pi$　　$\sin(x+\pi) = -\sin x$

$= \sin\left(\theta+\frac{\pi}{2}\right) - \sin\theta - \sin\left(\theta+\frac{\pi}{2}\right) + \sin\theta$

$= 0$

②解）$\alpha = \frac{\pi}{9}$ より　$9\alpha = \pi$

$5\alpha = \pi - 4\alpha$　　$7\alpha = \pi - 2\alpha$

（与式）$= \cos 2\alpha + \cos 4\alpha + \cos(\pi - 4\alpha) + \cos(\pi - 2\alpha)$

$\cos(\pi-\theta) = -\cos\theta$

$= \cos 2\alpha + \cos 4\alpha - \cos 4\alpha - \cos 2\alpha$

$= 0$