

$$\begin{aligned}
 &= \frac{(\sqrt{7}-\sqrt{5})+(\sqrt{7}+\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} \\
 &= \frac{2\sqrt{7}}{2} = \sqrt{7}
 \end{aligned}$$

$$(2) \quad xy = \frac{1}{\sqrt{7}+\sqrt{5}} \cdot \frac{1}{\sqrt{7}-\sqrt{5}} = \frac{1}{2}$$

$$(3) \quad x^2 + y^2 = (x+y)^2 - 2xy = (\sqrt{7})^2 - 2 \cdot \frac{1}{2} = 6$$

$$(4) \quad x^2y + xy^2 = xy(x+y) = \frac{1}{2} \cdot \sqrt{7} = \frac{\sqrt{7}}{2}$$

(p.31) 発展 練習 1

$$x = \frac{2}{\sqrt{5}+1} = \frac{2(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{\sqrt{5}-1}{2}$$

ゆえに

$$x+y = \frac{\sqrt{5}-1}{2} + \frac{\sqrt{5}+1}{2} = \sqrt{5}$$

$$xy = \frac{2}{\sqrt{5}+1} \cdot \frac{\sqrt{5}+1}{2} = 1$$

よって

$$\begin{aligned}
 x^3 + y^3 &= (x+y)^3 - 3xy(x+y) \\
 &= (\sqrt{5})^3 - 3 \cdot 1 \cdot \sqrt{5} \\
 &= 2\sqrt{5}
 \end{aligned}$$

(p.32) 発展 練習 1

$$(1) \quad \sqrt{7+2\sqrt{10}} = \sqrt{(5+2)+2\sqrt{5 \cdot 2}} = \sqrt{5} + \sqrt{2}$$

$$\begin{aligned}
 (2) \quad \sqrt{12-6\sqrt{3}} &= \sqrt{12-2\sqrt{27}} \\
 &= \sqrt{(9+3)-2\sqrt{9 \cdot 3}} \\
 &= \sqrt{9} - \sqrt{3} = 3 - \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad \sqrt{3+\sqrt{5}} &= \sqrt{\frac{6+2\sqrt{5}}{2}} = \frac{\sqrt{6+2\sqrt{5}}}{\sqrt{2}} \\
 &= \frac{\sqrt{(5+1)+2\sqrt{5 \cdot 1}}}{\sqrt{2}} \\
 &= \frac{\sqrt{5}+1}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{2}}{2}
 \end{aligned}$$